

# On Decomposition of Bitopological (1,2)\*-A- Continuity

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**Abstract:** - The aim of this paper is to give decompositions of continuity, namely (1,2)\*-A-continuity by providing the concepts of (1,2)\*-semi-continuity, (1,2)\*-C-continuity, (1,2)\*-β-continuity and (1,2)\*-LC-continuity.

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## 1. Introduction

To give a decomposition of continuity, Tong[13] introduced the notions of A-set and A-continuous mappings and proved that a map  $f : X \rightarrow Y$  is continuous if and only if it is both  $\alpha$ -continuous and A-continuous. Again, Tong[14] introduced the notions of B-sets and B-continuous mappings, and together with the notion of precontinuity he proved another decomposition of continuity i.e., A mapping  $f : X \rightarrow Y$  is continuous if and only if it is precontinuous and B-continuous. Ganster and Reilly[5] established a decomposition of A-continuity i.e., a mapping  $f : X \rightarrow Y$  is A-continuous if and only if it is semi-continuous and LC-continuous.

In this paper, we obtain decompositions of bitopological (1,2)\*-A-continuous. In most of the occasions, our ideas are illustrated and substantiated by suitable examples.

## 2. Preliminaries

Throughout this paper,  $X$  and  $Y$  denote bitopological spaces  $(X, \tau_1, \tau_2)$  and  $(Y, \sigma_1, \sigma_2)$ , respectively, on which no separation axioms are assumed.

### Definition 2.1

Let  $S$  be a subset of  $X$ . Then  $S$  is called  $\tau_{1,2}$ -open [10] if  $S = A \cup B$ , where  $A \in \tau_1$  and  $B \in \tau_2$ .

The complement of  $\tau_{1,2}$ -open set is called  $\tau_{1,2}$ -closed.

### Definition 2.2

Let  $A$  be a subset of  $X$ .

(i) The  $\tau_{1,2}$ -closure of  $A$  [10], denoted by  $\tau_{1,2}\text{-cl}(A)$ , is defined by

$$\cap \{U : A \subseteq U \text{ and } U \text{ is } \tau_{1,2}\text{-closed}\};$$

(ii) The  $\tau_{1,2}$ -interior of  $A$  [10], denoted by  $\tau_{1,2}\text{-int}(A)$ , is defined by

$$\cup \{U : U \subseteq A \text{ and } U \text{ is } \tau_{1,2}\text{-open}\}.$$

### Remark 2.3 [10]

Notice that  $\tau_{1,2}$ -open sets need not necessarily form a topology.

Now we recall some definitions and results, which are used in this paper.

#### Definition 2.4

A subset  $A$  of  $X$  is said to be

- (i)  $(1,2)^*$ -semi-open [10] if  $A \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))$ ,
- (ii)  $(1,2)^*$ -preopen [10] if  $A \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$ ,
- (iii)  $(1,2)^*$ - $\beta$ -open [12] if  $A \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)))$ ,
- (iv)  $(1,2)^*$ - $\alpha$ -open [10] if  $A \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)))$ ,
- (v) regular  $(1,2)^*$ -open [10] if  $A = \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$ .

The complements of the above- mentioned open sets are called their respective closed sets.

The family of all  $(1,2)^*$ -semi-open (resp.  $(1,2)^*$ -preopen,  $(1,2)^*$ - $\alpha$ -open,  $(1,2)^*$ - $\beta$ -open, regular  $(1,2)^*$ -open) sets of  $X$  will be denoted by  $(1,2)^*\text{-SO}(X)$  (resp.  $(1,2)^*\text{-PO}(X)$ ,  $(1,2)^*\text{-}\alpha\text{O}(X)$ ,  $(1,2)^*\text{-}\beta\text{O}(X)$ ,  $(1,2)^*\text{-RO}(X)$ ).

The  $(1,2)^*$ -preclosure,  $(1,2)^*\text{-pcl}(A)$ , of a subset  $A$  is the intersection of all  $(1,2)^*$ -preclosed subsets of  $X$  that contain  $A$ .

#### Example 2.5

Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, X, \{a\}\}$  and  $\tau_2 = \{\emptyset, X, \{c\}\}$ . Then the sets in  $\{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$  are  $\tau_{1,2}$ -open and the sets in  $\{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$  are  $\tau_{1,2}$ -closed.

#### Definition 2.6

A subset  $S$  of  $X$  is said to be

- (i) a  $(1,2)^*$ -A-set [10] if  $S = G \cap R$ , where  $G$  is  $\tau_{1,2}$ -open and  $R$  is regular  $(1,2)^*$ -closed,
- (ii) a  $(1,2)^*$ -t-set [10] if  $\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(S)) = \tau_{1,2}\text{-int}(S)$ ,

- (iii) a  $(1,2)^*$ -B-set [10] if  $S = G \cap R$ , where  $G$  is  $\tau_{1,2}$ -open and  $R$  is a

$(1,2)^*$ -t-set,

- (iv) a locally  $(1,2)^*$ -closed [9] if  $S = G \cap R$ , where  $G$  is  $\tau_{1,2}$ -open and  $R$  is  $\tau_{1,2}$ -closed.

The family of all  $(1,2)^*$ -A-sets (resp. locally  $(1,2)^*$ -closed sets,  $(1,2)^*$ -B-sets) of  $X$  will be denoted by  $(1,2)^*\text{-A}(X)$  (resp.  $(1,2)^*\text{-LC}(X)$ ,  $(1,2)^*\text{-B}(X)$ ).

The following Proposition is a direct consequence of the definition of  $(1,2)^*$ -t-sets.

#### Proposition 2.7

A subset  $A$  of a space  $X$  is a  $(1,2)^*$ -t-set if and only if it is  $(1,2)^*$ -semi-closed.

#### Proof

Let  $A$  be a  $(1,2)^*$ -semi-closed set. Then  $\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)) \subseteq A$ . Therefore  $\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)) \subseteq \tau_{1,2}\text{-int}(A)$ . We know that  $\tau_{1,2}\text{-int}(A) \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$ . Hence  $\tau_{1,2}\text{-int}(A) = \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$ . Then  $A$  is  $(1,2)^*$ -t-set.

Conversely, let  $A$  be a  $(1,2)^*$ -t-set. Then  $\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)) = \tau_{1,2}\text{-int}(A)$ . We have  $\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)) \subseteq \tau_{1,2}\text{-int}(A) \subseteq A$ . Hence  $\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)) \subseteq A$ . Therefore  $A$  is  $(1,2)^*$ -semi-closed set.

From the definitions, we can see  $(1,2)^*\text{-LC}(X) \subseteq (1,2)^*\text{-B}(X)$ .

#### Example 2.8

Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, X, \{a\}\}$  and  $\tau_2 = \{\emptyset, X\}$ . We have  $(1,2)^*\text{-LC}(X) = \{\emptyset, X, \{a\}, \{b, c\}\}$  and  $(1,2)^*\text{-B}(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{b, c\}\}$ . Clearly  $\{b\}$  is  $(1,2)^*\text{-B}$ -set but it is not locally  $(1,2)^*$ -closed.

**Remark 2.9 [10]**

- (i) A  $(1,2)^*$ -A-set is a  $(1,2)^*$ -B-set but not conversely.
- (ii) Every regular  $(1,2)^*$ -open set is  $\tau_{1,2}$  - open but not conversely.

**Proposition 2.10**

Let A be an  $\tau_{1,2}$ -open subset of a space X. Then  $\tau_{1,2}\text{-cl}(A)$  is regular  $(1,2)^*$ -closed.

**Proof**

Clearly,  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))) \subseteq \tau_{1,2}\text{-cl}(A)$ . So we need only to show that  $\tau_{1,2}\text{-cl}(A) \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)))$ . Now, from  $A \subseteq \tau_{1,2}\text{-cl}(A)$ , we have  $A \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$ . Therefore,  $\tau_{1,2}\text{-cl}(A) \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)))$ .

**Proposition 2.11 [11]**

Let A be a subset of a space X. Then  $(1,2)^*\text{-pcl}(A) = A \cup \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))$ .

**Remark 2.12 [9]**

A subset S of X is locally  $(1,2)^*$ -closed if and only if  $S = U \cap \tau_{1,2}\text{-cl}(S)$ , where U is  $\tau_{1,2}$  -open.

**Definition 2.13 [10, 12]**

Let  $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a map. Then f is said to be  $(1,2)^*$ -semi-continuous if  $f^{-1}(G) \in (1,2)^*\text{-SO}(X)$  for each  $\sigma_{1,2}$ -open set G of Y.

*The  $(1,2)^*$ - $\beta$ -continuity and  $(1,2)^*$ -A-continuity are analogously defined.* **Remark 2.14 [10]**  $(1,2)^*$ -A-sets and  $(1,2)^*$ -semi-open sets are independent.

Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, X, \{a\}, \{b, c\}\}$  and  $\tau_2 = \{\emptyset, X, \{b\}, \{a, c\}\}$ . Then the sets in  $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$  are  $\tau_{1,2}$ -open and the sets in  $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are  $\tau_{1,2}$ -closed.

We have  $\{c\}$  is  $(1,2)^*$ -A-set but not  $(1,2)^*$ -semi-open.

Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, X, \{a\}\}$  and  $\tau_2 = \{\emptyset, X\}$ . Then the sets in  $\{\emptyset, X, \{a\}\}$  are  $\tau_{1,2}$ -open and the sets in  $\{\emptyset, X, \{b, c\}\}$  are  $\tau_{1,2}$ -closed. We have  $\{a, b\}$  is not  $(1,2)^*$ -A-set but it is  $(1,2)^*$ -semi-open.

### 3. PROPERTIES OF BITOPOLOGICAL $(1,2)^*$ -SETS

In this section, we provide three theorems concerning decompositions of bitopological  $(1,2)^*$ -A-continuity. In the second theorem, a notion of  $(1,2)^*$ -C-sets which is weaker than that of locally  $(1,2)^*$ -closed sets is used.

**Definition 3.1**

A subset S of a space X is called  $(1,2)^*$ -C-set if  $S = G \cap R$ , where G is  $\tau_{1,2}$ -open and R is a  $(1,2)^*$ -preclosed.

**Remark 3.2**

- (i) The family of all  $(1,2)^*$ -C-sets of X will be denoted by  $(1,2)^*\text{-C}(X)$ .
- (ii) Every  $\tau_{1,2}$ -open set is  $(1,2)^*$ -C-set.
- (iii) Every  $(1,2)^*$ -preclosed set is  $(1,2)^*$ -C-set.

**Remark 3.3**

By definition 3.1, it is clear that  $(1,2)^*\text{-A}(X) \subseteq (1,2)^*\text{-LC}(X) \subseteq (1,2)^*\text{-C}(X)$ .

The following example shows that a  $(1,2)^*$ -C-set need not be a locally  $(1,2)^*$ -closed set and a locally  $(1,2)^*$ -closed set need not be a  $(1,2)^*$ -A-set.

**Example 3.4**

Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, X, \{a, b\}\}$  and  $\tau_2 = \{\emptyset, X, \{b, c\}\}$ . Then  $(1,2)^*\text{-A}(X) = \{\emptyset, X, \{a, b\}\}$ ,

$\{b, c\}\}$ ;  $(1,2)^*\text{-LC}(X) = \{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{b, c\}\}$  and  $(1,2)^*\text{-C}(X) = P(X)$ , where  $P(X)$  is the power set of  $X$ . Clearly,  $\{b\}$  is  $(1,2)^*\text{-C}$ -set but it is not locally  $(1,2)^*\text{-closed}$ . Moreover,  $\{a\}$  is locally  $(1,2)^*\text{-closed}$  but it is not  $(1,2)^*\text{-A}$ -set.

### Definition 3.5

A bitopological space  $(X, \tau_1, \tau_2)$  equipped with the family of all  $\tau_{1,2}$ -open sets will be called DRT-space if  $\text{int}_{\tau_1}(S) = \text{int}_{\tau_2}(S)$  for each  $\tau_{1,2}$ -closed subset  $S$  of  $X$ .

Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, X, \{a\}, \{b, c\}\}$  and  $\tau_2 = \{\emptyset, X, \{b\}, \{a, c\}\}$ . Then  $(X, \tau_1, \tau_2)$  is not DRT-space since  $\text{int}_{\tau_1}(\{a\}) = \{a\} \neq \emptyset = \text{int}_{\tau_2}(\{a\})$  for the  $\tau_{1,2}$ -closed subset  $\{a\}$  of  $X$ . However, in Example 3.4.,  $(X, \tau_1, \tau_2)$  is DRT-space.

### Theorem 3.6

Let  $X$  be a DRT-space. Then an  $(1,2)^*\text{-A}$ -set in  $X$  is  $(1,2)^*\text{-semi-open}$ .

#### Proof

Let  $S = U \cap C$  be an  $(1,2)^*\text{-A}$ -set, where  $U$  is  $\tau_{1,2}$ -open and  $C = \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(C))$ . Since  $S = U \cap C$ , we have  $\tau_{1,2}\text{-int}(S) \supset U \cap \tau_{1,2}\text{-int}(C)$ . It is easily seen that  $\tau_{1,2}\text{-int}(S) \subset S \subset C$ , hence  $\tau_{1,2}\text{-int}(S) = \tau_{1,2}\text{-int}(\tau_{1,2}\text{-int}(S)) \subset \tau_{1,2}\text{-int}(C)$ . But  $\tau_{1,2}\text{-int}(S) \subset S \subset U$ , hence  $\tau_{1,2}\text{-int}(S) \subset U \cap \tau_{1,2}\text{-int}(C)$ . Therefore  $\tau_{1,2}\text{-int}(S) = U \cap \tau_{1,2}\text{-int}(C)$ . Now we prove  $S \subset \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(S))$ . Let  $x \in S$  and  $V$  be an arbitrary  $\tau_{1,2}$ -open set containing  $x$ . Then  $U \cap V$  is also an  $\tau_{1,2}$ -open set containing  $x$ . Since  $x \in C = \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(C))$ , there is a point  $z \in \tau_{1,2}\text{-int}(C)$  such that  $z \neq x$  and  $z \in U \cap V$ . Hence  $z \in U \cap \tau_{1,2}\text{-int}(C) = \tau_{1,2}\text{-int}(S)$ . Therefore  $x \in \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(S))$  and  $S \subset \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(S))$ .

From  $\tau_{1,2}\text{-int}(S) \subset S \subset \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(S))$  we know that  $S$  is  $(1,2)^*\text{-semi-open}$ .

### Example 3.7

Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, X, \{a\}\}$  and  $\tau_2 = \{\emptyset, X\}$ . Then the sets in  $\{\emptyset, X, \{a\}\}$  are  $\tau_{1,2}$ -open and the sets in  $\{\emptyset, X, \{b, c\}\}$  are  $\tau_{1,2}$ -closed. In this DRT-space, we have  $\{a, b\}$  is not  $(1,2)^*\text{-A}$ -set but it is  $(1,2)^*\text{-semi-open}$ .

### Theorem 3.8

Let  $X$  be a DRT-space. Then  $(1,2)^*\text{-A}(X) = (1,2)^*\text{-SO}(X) \cap (1,2)^*\text{-LC}(X)$ .

#### Proof

Let  $S \in (1,2)^*\text{-A}(X)$ . Then  $S = G \cap R$  where  $G$  is  $\tau_{1,2}$ -open and  $R$  is regular  $(1,2)^*\text{-closed}$ . Clearly  $S$  is locally  $(1,2)^*\text{-closed}$ . Now  $\tau_{1,2}\text{-int}(S) = G \cap \tau_{1,2}\text{-int}(R)$ , so that  $S = G \cap \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(R)) \subseteq \tau_{1,2}\text{-cl}(G \cap \tau_{1,2}\text{-int}(R)) = \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(S))$  and hence  $S$  is  $(1,2)^*\text{-semi-open}$ .

Conversely, let  $S$  be  $(1,2)^*\text{-semi-open}$  and locally  $(1,2)^*\text{-closed}$ , so that  $S \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(S))$  and  $S = U \cap \tau_{1,2}\text{-cl}(S)$ , where  $U$  is  $\tau_{1,2}$ -open. Then  $\tau_{1,2}\text{-cl}(S) = \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(S))$  and so is regular  $(1,2)^*\text{-closed}$ . Hence  $S$  is an  $(1,2)^*\text{-A}$ -set.

### Proposition 3.9

Let  $A$  be a subset of  $X$ . Then  $A$  is  $(1,2)^*\text{-preclosed}$  if and only if  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq A$ .

### Lemma 3.10

Let  $(X, \tau_1, \tau_2)$  be a DRT-space and  $G$  be a subset of  $X$ . Then  $G \in (1,2)^*\text{-C}(X)$  if and only if  $G = R \cap (1,2)^*\text{-pcl}(G)$  for some  $\tau_{1,2}$ -open set  $R$ .

### Proof

Suppose that  $G = R \cap (1,2)^*\text{-pcl}(G)$  for some  $\tau_{1,2}$ -open set  $R$ . It is obvious that  $G \in (1,2)^*\text{-C}(X)$ , since  $(1,2)^*\text{-pcl}(G)$  is  $(1,2)^*\text{-preclosed}$ .

Conversely, let  $G \in (1,2)^*\text{-C}(X)$ . Then  $G = R \cap A$  where  $R$  is  $\tau_{1,2}$ -open and  $A$  is  $(1,2)^*\text{-preclosed}$ . From  $G \subseteq A$ , we have  $(1,2)^*\text{-pcl}(G) \subseteq (1,2)^*\text{-pcl}(A) = A \cup \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))$ . Since  $A$  is  $(1,2)^*\text{-preclosed}$ , by Proposition 3.9, we have  $(1,2)^*\text{-pcl}(G) \subseteq A$ . Thus,  $R \cap (1,2)^*\text{-pcl}(G) \subseteq R \cap A = G \subseteq R \cap (1,2)^*\text{-pcl}(G)$ , which shows that  $G = R \cap (1,2)^*\text{-pcl}(G)$  with  $R$  is  $\tau_{1,2}$ -open.

### Lemma 3.11

Let  $(X, \tau_1, \tau_2)$  be a DRT-space and  $G$  be a subset of  $X$ . Then  $G = R \cap \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(G))$  for some  $\tau_{1,2}$ -open set  $R$  if and only if  $G \in (1,2)^*\text{-C}(X) \cap (1,2)^*\text{-SO}(X)$ .

### Proof

Suppose that  $G = R \cap \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(G))$  where  $R$  is  $\tau_{1,2}$ -open. Then  $G \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(G))$  which shows that  $G \in (1,2)^*\text{-SO}(X)$ . Moreover,  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(G))$  is  $\tau_{1,2}$ -closed and therefore  $(1,2)^*\text{-preclosed}$ . So,  $G \in (1,2)^*\text{-C}(X)$ .

Conversely, let  $G \in (1,2)^*\text{-C}(X) \cap (1,2)^*\text{-SO}(X)$ . From  $G \in (1,2)^*\text{-C}(X)$ , we have from Lemma 3.10, that  $G = R \cap (1,2)^*\text{-pcl}(G)$ , where  $R$  is  $\tau_{1,2}$ -open. From  $G \in (1,2)^*\text{-SO}(X)$ , we have  $G \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(G))$ . But  $(1,2)^*\text{-pcl}(G) = G \cup \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(G))$  (see Proposition 2.11). Thus  $G = R \cap \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(G))$  with  $R$  is  $\tau_{1,2}$ -open.

### Theorem 3.12

Let  $(X, \tau_1, \tau_2)$  be a DRT-space. Then  $(1,2)^*\text{-A}(X) = (1,2)^*\text{-C}(X) \cap (1,2)^*\text{-SO}(X)$ .

### Proof

It is clear that  $(1,2)^*\text{-A}(X) \subseteq (1,2)^*\text{-C}(X) \cap (1,2)^*\text{-SO}(X)$ .

Conversely, let  $G \in (1,2)^*\text{-C}(X) \cap (1,2)^*\text{-SO}(X)$ . Then by Lemma 3.11,  $G = R \cap \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(G))$ , where  $R$  is  $\tau_{1,2}$ -open. Since  $\tau_{1,2}\text{-int}(G)$  is  $\tau_{1,2}$ -open, by Proposition 2.10,  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(G))$  is regular  $(1,2)^*\text{-closed}$ . Therefore  $G \in (1,2)^*\text{-A}(X)$ .

### Remark 3.13

It is clear from the definition 2.4 that  $(1,2)^*\text{-SO}(X) \subseteq (1,2)^*\text{-}\beta\text{O}(X)$ . However, the converse is not true.

### Example 3.14

Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, X, \{a\}\}$  and  $\tau_2 = \{\emptyset, X, \{b, c\}\}$ . Then the sets in  $\{\emptyset, X, \{a\}, \{b, c\}\}$  are  $\tau_{1,2}$ -open and  $\tau_{1,2}$ -closed. Clearly,  $\{b\} \in (1,2)^*\text{-}\beta\text{O}(X)$ , but it is not  $(1,2)^*\text{-semi-open}$ .

### Theorem 3.15

Let  $(X, \tau_1, \tau_2)$  be a DRT-space. Then  $(1,2)^*\text{-A}(X) = (1,2)^*\text{-}\beta\text{O}(X) \cap (1,2)^*\text{-LC}(X)$ .

### Proof

If  $G \in (1,2)^*\text{-A}(X)$ , then, obviously,  $G \in (1,2)^*\text{-}\beta\text{O}(X) \cap (1,2)^*\text{-LC}(X)$ . Conversely, let  $G \in (1,2)^*\text{-}\beta\text{O}(X) \cap (1,2)^*\text{-LC}(X)$ . From  $G \in (1,2)^*\text{-}\beta\text{O}(X)$ , we have  $G \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(G)))$ . From  $G \in (1,2)^*\text{-LC}(X)$ , we have, by Remark 2.12,  $G = U \cap \tau_{1,2}\text{-cl}(G)$ , where  $U$  is  $\tau_{1,2}$ -open. So  $G \subseteq U$ , which implies  $G \subseteq U \cap \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(G))) \subseteq U \cap \tau_{1,2}\text{-cl}(G) = G$ . Hence we have  $G = U \cap \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(G)))$ . By Proposition 2.10,  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(G)))$  is regular  $(1,2)^*\text{-closed}$ , since  $\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(G))$  is  $\tau_{1,2}$ -open. Therefore,  $G \in (1,2)^*\text{-A}(X)$ .

#### 4. DECOMPOSITIONS OF $(1,2)^*$ -A-CONTINUITY

##### Definition 4.1

A mapping  $f : X \rightarrow Y$  is said to be  $(1,2)^*$ -continuous [9] if  $f^{-1}(V)$  is  $\tau_{1,2}$ -open in  $X$  for every  $\sigma_{1,2}$ -open set  $V$  of  $Y$ .

##### Definition 4.2

A mapping  $f : X \rightarrow Y$  is said to be  $(1,2)^*$ -C-continuous if  $f^{-1}(V) \in (1,2)^*\text{-C}(X)$  for every  $\sigma_{1,2}$ -open set  $V$  of  $Y$ .

##### Definition 4.3

A mapping  $f : X \rightarrow Y$  is said to be  $(1,2)^*$ -LC-continuous [9] if  $f^{-1}(V) \in (1,2)^*\text{-LC}(X)$  for every  $\sigma_{1,2}$ -open set  $V$  of  $Y$ .

##### Theorem 4.4

Let  $(X, \tau_1, \tau_2)$  be a DRT-space. Then a mapping  $f : X \rightarrow Y$  is  $(1,2)^*$ -A-continuous if and only if it is  $(1,2)^*$ -semi-continuous and  $(1,2)^*$ -C-continuous.

##### Proof

It follows from Theorem 3.12.

##### Theorem 4.5

Let  $(X, \tau_1, \tau_2)$  be a DRT-space. Then a mapping  $f : X \rightarrow Y$  is  $(1,2)^*$ -A-continuous if and only if it is  $(1,2)^*$ -semi-continuous and  $(1,2)^*$ -LC-continuous.

##### Proof

It follows from Theorem 3.8.

##### Theorem 4.6

Let  $(X, \tau_1, \tau_2)$  be a DRT-space. Then a mapping  $f : X \rightarrow Y$  is  $(1,2)^*$ -A-continuous if and only

if it is  $(1,2)^*$ - $\beta$ -continuous and  $(1,2)^*$ -LC-continuous.

##### Proof

It follows from Theorem 3.15.

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